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$$\begin{array}{lll}
\eta_1{}'=\rho_1\eta_1, & \eta_5{}'=\rho_1\eta_5+\rho_5\eta_1, & \\
\eta_2{}'=\rho_1\eta_2, & \eta_6{}'=\rho_1\eta_6+\rho_6\eta_1 & +\rho_5\eta_2, \\
\eta_3{}'=\rho_1\eta_3, & \eta_7{}'=\rho_1\eta_7+\rho_7\eta_1 & +\rho_5\eta_2+\rho_5\eta_3, \\
\eta_4{}'=\rho_1\eta_4, & \eta_8{}'=\rho_1\eta_8+\rho_8\eta_1 & +(\rho_5+\rho_6+\rho_7)\eta_2+\rho_6\eta_3+\rho_5\eta_4.
\end{array}$$

Denoting this transformation by T_ρ , we verify that $T_\rho\ T_\rho = T_{\rho''}$, where

$$\rho_1{}''=\rho_1\rho_1{}',\ \rho_5{}''=\rho_5\rho_1{}'+\rho_1\rho_5{}',\ \rho_6{}''=\rho_6\rho_1{}'+\rho_1\rho_6{}',$$
$$\rho_7{}''=\rho_7\rho_1{}'+\rho_1\rho_7{}',\ \rho_8{}''=\rho_8\rho_1{}'+\rho_1\rho_8{}'.$$

Hence the transformations T_ρ form a group and the group is commutative. As this result is in accord with the general theory, we have a complete check upon the above work.

The University of Chicago, October 10, 1902.

THE LENGTH OF THE CIRCLE.

By DR. GEORGE BRUCE HALSTED.

The simple theorem of Euclid (Eu. I. 20) that any two sides of a triangle are together greater than the third, appears in popular geometries as a result, a deduction from the definition: the straight line is the shortest distance between two points. But such a definition is, as Hilbert says, *senseless* if one has not defined the concept length.

Without presupposing the idea of the length of the curve, it cannot use anything but the straight, and so is merely a *petitio principii.*

In fact, as Hilbert points out, the essential content of the statement, the straight line is the shortest distance between two points, reduces merely to this theorem of Euclid.

The same fallacy lurks in the theorem, "An arc of a circle is less than any line which envelops it and has the same extremities," recently 'borrowed' by Wentworth (1899, p. 219, Book V, Proposition VII, §451) from Chauvenet (1869, 1881, p. 155, Book V, Proposition XII, §32). He says "Of all the lines, etc., there must be at least one shortest line."

But what is the length of a curve? Again we have a begging of the question. Upon these two holes, empty places, or chasms, he then attempts to rest a demonstration that the length of a circle is the limit of the lengths of the perimeters of inscribed polygons.

The sooner this procedure is banished from elementary geometry the better for the world.

Fortunately there is here no need to touch that very difficult general question, what is the length of a curve in general.

We have only for consideration the simplest of curves, the circle.

For it then only need we show the simple theorem (Halsted's *Mensuration*, p. 16): When their sides tend indefinitely towards zero, the perimeter of the polygon inscribed increases, circumscribed decreases, toward the same limit.

After that, we may deliberately and consciously declare by *definition* that this limit shall be what we will mean when we use the word *length* in connection with *circle.*

Here is no attempt to *prove* that the length of the circle is the limit of the lengths of perimeters of polygons. Any such attempt presupposes that we already know in some other way, or have in some other mathematical way defined what we are then already to mean by the length of the circle before we try to prove it equivalent to the limit for perimeters. What text book does this?

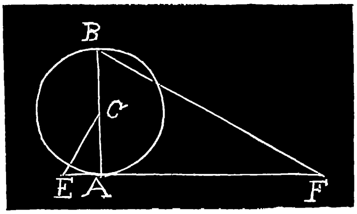
Can it be done? Let me try. (*Sect* is English for the German *strecke*).

Definition of length of an arc. We assume that with every arc is connected a sect such that if an arc be cut into two arcs, this sect is the sum of their sects; moreover this sect is not less than the chord of the arc, nor, if the arc be minor, is it greater than the sum of the sects on the tangents from the extremities of the arc to their intersection.

This sect we call *the length of the arc.*

Kochansky (1685) gave the following simple construction for the approximate length of the semicircle.

At the end-point A of the diameter BA, draw the tangent to the circle with center C. Take $ACE==\frac{1}{3}$ right angle. On the tangent, take $EF=3AC$. Then BF is with great exactitude the length of the semicircle.



In fact, $BF=r[13\frac{1}{2}-2(3)^{\frac{1}{2}}]^{\frac{1}{2}}=r3.1415$.

University of Texas, November, 1902.

AN ELEMENTARY ACCOUNT OF THE PICARD-VESSIOT THEORY.

By DR. SAUL EPSTEEN.

A theory of linear differential equations has been built up within the last twenty years which resembles the Galois theory of algebraic equations very closely.

In the Galois theory of equations we begin with an algebraic equation

$$(1)\qquad x^n+a_1x^{n-1}+\ldots.+a_n=0$$

and study the n! permutations of the n roots x_1, ..., x_n.